UNIT 1

# Introduction, set notation

* Sets can be finite or infinite. Signified by {}.
* Things inside a set are called *elements*
* Redundancies are removed i.e. {1,3,2,1,3,4…} = {1,2,3,4…}

Example 1

B = {2,4,6,8,10}

* The size (or cardinality) of B is 5 or CARD(B) = 5
* 4 is an element of B/ 4 is a member of B B
* 12 is not an element of B B

= {}

* The size (or cardinality) of the set is 0
* This is the empty set (NOT = { })

Example 2

The set of even integers

* The set of n such that n is even

(If a variable is even, write it in terms of another variable as a set of even numbers.

E.g. if m is even, then, )

The set of Prime numbers

* The set of n such that n is prime

OR

“Such that”

The set of Prime numbers

* The set of n such that n is prime

The set of odd numbers

* The set of n such that n is odd

# 1.2 . Positive Integers

* Set of all positive integers

= {1,2,3,…} Positive integers

## 1.2.1 Commutative property

* For the set of , the numbers m and m, addition and multiplication is commutative

“Order does not matter”.

N.B. ALWAYS write using brackets for products. This is because (m)(n) ≠ m.n i.e. (3)(-4) = (-4)(-3) ≠ -3

(m)(n) = (n)(m) Multiplicative Commutative Property

m + n = n + m Additive Commutative Property

*Commutative = Substitution or Exchange*

## 1.2.2 Associative Property

“Add or multiply regardless of how the numbers are grouped”.

* For the set of , the numbers m, n and k addition and multiplication are associative

*It’s like whether you add a number to the beginning or end of a sum, the total remains the same*

*(The grouping of adding does not change the sum, BODMAS)*

(m)(nk) = (mn)(k)

m + (n + k) = (m + n) + k

## 1.2.3 Distributive Property

* For the set of , the numbers m, n and k are distributive over addition

m(n + k) = mn + mk

(n + k)m = m(n + k)

= mn + mk by distributivity

Swap around n and m

= nm + km by commutativity

*It’s like taking out a common factor* “mn + mk is equal to m(n + k)”

## 1.2.4 Multiplicative identity

* 1 (the multiplicative identity element)
* for every , m, m⋅1 = m

# 1.3 Non-negative Integers

* set of all non-negative integers

OR = {0,1,2,3,…} Non-negative integers (additional number zero.)

## 1.3.1 Additive Identity

* for every non-negative integer m, m + 0 = m

*This is a tool we use to add to both sides of the equation to make it easier to solve. This is REALLY helpful because we can even add square numbers to both sides*

## 1.3.2 Multiplication by zero

* for every non-negative integer m, m⋅0 = 0

# 1.4 Integers

* This topic can be a little bit more complicated, some maths symbols (such as ) are for non-negative numbers, and don’t make use of negative numbers.
* Adding is simple: adding numbers can only give one result, use a number scale
* Multiplication is complicated. There are four different possibilities for a product (x)(y)

1. If x and y are , then the product =
2. If x is , then the product is (repeated addition)
3. Multiplication of integers is NOT commutative. i.e. the order matters when multiplying.
4. If x AND y are, then;
   1. If x is negative, then x = −a for some positive integer a.
   2. If y is negative, then y = −b for some positive integer b.
5. Otherwise y = b. If one of x and y is negative, then xy = −(ab).

Otherwise, xy = ab.

# 1.5 The additive inverse, absolute values and prime numbers

## Additive Inverse

* Remember that – x DOES NOT always denote a negative number
* Therefore, read –x as *“*− *x is the additive inverse of x”.*
* If you think of x as representing a certain number of steps in either the positive or the negative direction, then −x represents an equal number of steps in the opposite direction.
* Using the additive inverse helps solve equations in the form (remember quadratic equations have two answers, one positive, one negative).
* So think of it like this:

So if x = −2, for instance,

then −x is the additive inverse of −2,

i.e. −x = −(−2) = 2.

*This is just a complicated of way of speaking of the negative sign*

*ALSO used to get rid of absolute value signs*

## Absolute Value

* x if x is non-negative, or −x if x is negative.
* Very easy, value of a number in an absolute value sign is always positive.
* Therefore, and

Remember, , since 5 is further away from zero than 2

## Prime numbers

* Number that is divisible by 1 and itself

## Factorial (LEARN WITH COMBINATIONS)

* the **factorial** of a [non-negative integer](https://en.wikipedia.org/wiki/Non-negative_integer) *n*, denoted by *n*!, is the [product](https://en.wikipedia.org/wiki/Product_(mathematics)) of all positive [integers](https://en.wikipedia.org/wiki/Integer) less than or equal to *n*.
* Therefore
* For example,
* Although,

Factorial shorthand:

Or

Because 4!, implies that we’re still counting down till 1

# 1.6 The nine laws for Z≥

*C. A. M. E. L. T. E. A. D*

*2. 3. 3. 1. 2. 3. 1. 1. 3*

*Law 1 (commutativity):*

For all non-negative integers m and n,

m + n = n + m and mn = nm.

*Law 2 (associativity):*

For all non-negative integers m, n and k,

m + (n + k) = (m + n) + k and m(nk) = (mn)k.

*Law 3 (distributivity):*

For all non-negative integers m, n and k,

m(n+k) = (mn) + (mk).

*Law 4 (existence of a multiplicative identity element):*

For all non-negative integers m,

m⋅1 = m.

*Law 5 (linearity):*

For all non-negative integers m and n, exactly one of the following

statements are true:

m < n, m = n, m > n.

*Law 6 (monotonicity of + and* × respectively*):*

For all non-negative integers m, n and k,

if m = n, then m + k = n + k and mk = nk;

if m < n, then m + k < n + k; and

if k > 0, mk < nk.

*Law 7 (transitivity of = and < respectively):* i.e Transitive Property

For all non-negative integers m, n and k,

if m = n and n = k, then m = k, and

if m < n and n < k, then m < k.

*Law 8 (existence of an additive identity element):*

For all non-negative integers m,

m + 0 = m.

*Law 9 (absence of zero-divisors):*

For all non-negative integers m and n,

mn = 0 if and only if m = 0 or n = 0.

Number of variables in notation